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Similarity Solutions for the Flow of A Mixture of A Non-Ideal Gas and Small Solid Particles Behind an Exponential Shock in Magnetogasdynamics

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ABSTRACT: A self-similar method is used to analyse numerically the one-dimensional, unsteady, self-similar flow of a perfectly conducting mixture of a non-ideal gas and small solid particles, behind a strong shock, driven by a cylindrical or spherical piston, moving according to an exponential law in the presence of an azimuthal magnetic field. In order to get some essential features of shock propagation the small solid particles are considered as pseudo fluid. It is also assumed that the equilibrium flow condition is maintained in the whole flow- field and the viscous stress and heat conduction of the mixture are negligible. Effects of change in the values of parameters Kp, G_1 (dust parameters), b^{-}(non-idealness parameter of the gas) and M_A^{-2} (magnetic parameter) on the shock strength, piston position and on the flow-variables in the flow-field behind the shock front are obtained. It is found that there is a decrease in the shock strength and the value of piston position due to non-idealness of the gas as well as due to the presence of dust-particles and the magnetic-field. This decrease in the shock strength and the value of piston position is interpreted as a result of decrease in the compressibility of the mixture. Mutual effects of parameters Kp, G_1 , $b^{-}$and M_A^{-2} are also obtained and it is found that magnetic-field slightly reduces the effects of Kp , G_1 and b^{-}on the shock strength and on the piston position; whereas presences of dust particles significantly reduce the effects of magnetic field on shock strength, piston position and on the the flow-variables. Solutions are obtained for both the isothermal and adiabatic flows of the mixture. A comparative study is also made between these two flows of the mixture and is found that assumption of zero temperature gradient brings a profound change in the density profile as compared to that of adiabatic flow whereas profiles of the other flow-variables are little affected.

KEYWORDS: Shock waves, piston problem, self-similar solutions, two-phase flow, non-ideal gas, small-solid particles, magnetic-field, exponential shock, isothermal flow, adiabatic flow.

1. INTRODUCTION:

The problem of shock wave propagation in two-phase flow has been studied by many research workers such as Hiagashino and Suzuki [1], Pai et al. [2] Miura and Glass [3], Higashino [4], Vishwakarma and Nath [5] and Nath [6] etc., because of its application in variety of fields such as astrophysics, geophysics, nuclear science, plasma physics etc. Sedov [7] and Ranga Rao and Ramana [8] indicated that a limiting case of a self-similar flow with power law shock is a flow-field formed with an exponential law. Ranga Rao and Ramana [8] and Singh and Vishwakarma [9] have obtained solutions for the problem of unsteady, self-similar motion of a gas displaced by a piston according to an exponential law. Vishwakarma and Nath [5] have obtained the self-similar solutions for both the isothermal and adiabatic flows of a mixture of a perfect gas and small solid particles behind an exponential shock and discussed the effects of the presence of dust particles on the shock strength and on the flow variables.

In all the above-mentioned works, the dusty gas is assumed to be a mixture of perfect gas and small solid particles. But the perfect gas law cannot be applied to actual gas with sufficient accuracy when the interaction between its component molecules occurs. Therefore a number of study have been made for the shock wave propagation in a non-ideal gas, particularly, by Anisimov and Spiner [10], Ranga Rao and Purohit [11], Vishwakarma, Patel and Chaube [12], Vishwakarma and Nath [13], Vishwakarma and Patel [14], Vishwakarma and Nath [15], Ojha and Singh [16], Nath [17], Sahu [18] etc.. Vishwakarma and Nath [13] have obtained the similarity solution for the isothermal and adiabatic flows of a non-ideal gas behind an exponential shock. Vishwakarma and Patel [14] have extended this work by taking a mixture of non-ideal gas and small solid particles in place of non-ideal gas.

In the present work, we derive the similarity solution for the shock wave propagation in a magnetised non-ideal dusty gas. This work may have application to many astrophysical and nuclear science phenomena like motion in interstellar medium, supernova explosion, heliosphere etc. in which shock waves appear in the magnetised dusty medium. Some authors have discussed the shock propagation in magnetised non-ideal gas, particularly, Nath [19], Vishwakarma and Patel [20], Nath and Sahu [21], Singh et al. [22] etc.. Singh et al. have obtained the self-similar solution for exponential shock wave in non-ideal magnetogasdynamics. But no one has studied the problem of shock wave propagation in magnetised non-ideal dusty gas.

In this paper we investigate the self-similar solution for the propagation of shock wave driven out by a cylindrical or spherical piston moving according to an exponential law, namely,

$$r_p = A\exp(\lambda t),\ \lambda > 0, \tag{1}$$

where r_p is the radius of piston, A and λ are dimensional constants and t is the time.

Since motion is self-similar therefore shock and piston both obey same law. Hence shock will also be exponential. Thus,

$$R = B\exp(\lambda t), \tag{2}$$

In order to get some essential features of shock propagation, small solid particles are taken as pseudo-fluid and it is assumed that the equilibrium flow condition is maintained in the whole flow-field and the mixture is permeated by an azimuthal magnetic field. Also, the viscous stress and heat conduction of the mixture are assumed to be negligible.

The solutions are obtained in section when the flow is adiabatic. An alternative assumption of zero temperature gradient (isothermal flow) throughout the flow may also be taken (Korobeinikov [23], Laumbach and Probstein [24], Sachdev and Ashraf [25]), because when flow is in extreme conditions, transfer of heat takes place behind a strong shock by the mode of radiation and for this condition assumption of adiabaticity may not be valid.

Effects of a change in the values of parameters K_p and G_1(dust parameters), $\bar{b}$ (non-idealness parameter of the gas), M_A^{-2} (magnetic parameter) are obtained on the propagation of shock and on the flow-field behind it. A comparison is also made between the solutions of isothermal and adiabatic flows. An interaction of parameters K_p , G_1 and $\bar{b}$ with the parameter M_A^{-2} is obtained to find the changes in the effects of these parameters due to the presence of magnetic field.

2. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS: ISOTHERMAL FLOW-

The equations of motion for a one-dimensional, unsteady, isothermal flow of a perfectly conducting mixture of a non-ideal gas and small solid particles in the presence of an azimuthal magnetic field can be written as-

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{j\rho u}{r} = 0\,, \tag{3}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] = 0\,, \tag{4}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + (j-1)\frac{hu}{r} = 0\,, \tag{5}$$

$$\frac{\partial T}{\partial r} = 0\,, \tag{6}$$

where ρ is the density of the mixture, u is the flow velocity, p is the pressure of the mixture, h is the azimuthal magnetic field, μ is the magnetic permeability, r and t are the space and time co-ordinates and $j =$ 1or 2 for cylindrical and spherical symmetry.

We consider the medium to be a perfectly conducting mixture of a non-ideal gas and small solid particles permeated by an azimuthal magnetic field.

The equation of state of non-ideal gas in the mixture is taken to be (Vishwakarma and Nath [15], Ranga Rao and Purohit [11], Anisimov and Spiner [10])

$$p_g = R^* \rho'_g (1 + b\rho'_g) T, \tag{7}$$

where p_g and $\acute{\rho}_g$ are the partial pressure and partial density of the gas in the mixture, T is the temperature of the gas (and of the solid particles as the equilibrium flow condition is maintained), R^* is the specific gas constant and b is the internal volume of the gas. There is an interaction between component molecules of the non-ideal, this deviation of non-ideal gas from the ideal state is taken into account in the above equation. It is also assumed that the density of the non-ideal gas is so small that the triple, quadruple and higher order collisions among the molecules of the gas are negligible and therefore the gas molecules interact through binary collisions only.

The specific volume of solid particle is assumed to remain unchanged by variation in the temperature and pressure.

Thus, the equation of state of solid particles in the mixture is,

$$\rho_{sp} = \text{constant}, \qquad (8)$$

where ρ_{sp} is the species density of the solid particles.

The equation of state of the mixture may be written as (Vishwakarma and Nath [15]

$$p = \frac{1-K_p}{1-Z}\left[1 + b\rho\left(1-K_p\right)\right]\rho RT \qquad (9)$$

where $Z = \frac{V_{sp}}{V}$ is the volume fraction and $K_p = \frac{m_{sp}}{m}$ is the mass fraction (concentration) of the solid particles in the mixture. Here m_{sp} and V_{sp} are the total mass and volumetric extension of the solid particles and V and m are the total volume and total mass of the mixture.

The relation between K_p and Z is given by (Pai [26])

$$K_p = \frac{Z\,\rho_{sp}}{\rho}. \qquad (10)$$

In equilibrium flow, K_p is constant in whole flow field. Therefore from (10)

$$\frac{Z}{\rho} = \text{cconstant}. \qquad (11)$$

Also, we have the relation (Pai [26])

$$Z = \frac{K_p}{G(1-K_p)+K_p}, \qquad (12)$$

where $G = \frac{\rho_{sp}}{\rho_g}$ is the ratio of the density of the solid particles to the species density of the gas.

The internal energy per unit mass of the mixture may be written as

$$E_m = [K_p\, C_{sp} + (1-K_p)C_v\,]\,T = C_{vm}\,T, \qquad (13)$$

where C_{sp} is the specific heat of the solid particles, C_v is the specific heat of the gas at constant volume and C_{vm} is the specific heat of the mixture at constant volume.

The specific heat of the mixture at constant pressure is

$$C_{pm} = K_p C_{sp} + (1-K_p)C_p\,, \qquad (14)$$

where C_p is the specific heat of the gas at constant pressure.

The ratio of the specific heats of the mixture is given by (Pai et al. [2], Pai [26], Marble [27]),

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \gamma\,\frac{1+\delta\beta'/\gamma}{1+\delta\beta'}\,, \qquad (15)$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{K_p}{1-K_p}$, and $\beta' = \frac{C_{sp}}{C_v}$.

Now $C_{pm} - C_{vm} = \left(1-K_p\right)\left(C_p - C_v\right) =$

$$\left(1-K_p\right) \qquad (16)$$

where $R^* = \left(C_p - C_v\right)$, neglecting the term $b^2\rho^2$ (Anisimov and spiner [10], Singh [28]). The internal energy per unit mass of the mixture is, therefore, given by

$$E_m = \frac{p\,(1-Z)}{\rho\,(\Gamma-1)\,[1+b\rho\,(1-K_p)]}. \qquad (17)$$

2.1. Jump Conditions

Now we consider that a strong shock (cylindrical or spherical) driven by a piston moving according to an exponential law, is propagated into the perfectly conducting mixture of a non-ideal gas and small solid particles, of constant density ρ_1 at rest ($u_1 = 0$) and with negligibly small counter pressure ($p_1 = 0$), in the presence of an azimuthal magnetic field. The azimuthal magnetic field is varying as $h = Br^{-k}$, where B and k are constants.

The flow variables immediately ahead of the shock are

$$\rho = \rho_1 = \text{constant},$$

$$h = h_1 = bR^{-k}\,, \qquad (18)$$

$$p = p_1 = \frac{(1-k)\mu B^2}{2kR^{2k}}\,, \qquad 0 < k < 1\,,$$

where R is the radius of the shock and variables with subscript ′1′ denote their values immediately ahead of the shock.

From relation (12) we have

$$Z_1 = \frac{K_p}{G_1(1-K_p)+K_p}, \tag{19}$$

where $G_1 = \frac{\rho_{sp}}{\rho_{g_1}}$ is the ratio of the density of the solid particles to the initial density of gas.

Jump conditions across the strong shock front are as follows

$$\rho_2 (U-u_2) = \rho_1 U,$$

$$h_2(U-u_2) = h_1 U,$$

$$p_2 + \rho_2(U-u_2)^2 + \frac{\mu h_2^2}{2} = \rho_1 U^2 + \frac{\mu h_1^2}{2},$$

$$E_{m_2} + \frac{p_2}{\rho_2} + \frac{1}{2}(U-u_2)^2 + \frac{\mu h_2^2}{\rho_2} = \frac{U^2}{2} + \frac{\mu h_1^2}{\rho_1},$$

$$\frac{Z_2}{\rho_2} = \frac{Z_1}{\rho_1} \tag{20}$$

where subscript '2' refers to the values immediately behind the shock and $U = \frac{dR}{dt}$ is the shock velocity.

From shock conditions (20), we have

$$u_2 = (1-\beta)U,$$

$$p_2 = \left\{(1-\beta) + \frac{M_A^{-2}}{2}\left(1-\frac{1}{\beta^2}\right)\right\}\rho_1 U^2,$$

$$\rho_2 = \frac{1}{\beta}\rho_1, \quad Z_2 = \frac{1}{\beta}Z_1,$$

$$h_2 = (1-\beta)U, \tag{21}$$

where β ($0<\beta<1$) is given by the relation

$$(\Gamma+1)\beta^3 + \left[\left\{\bar{b}(1-K_p)-1\right\}(\Gamma-1) - 2Z_a - \Gamma M_A^{-2}\right]\beta^2 + \left[\left\{(Z_a-2) + \Gamma - \bar{b}(1-K_p)(\Gamma-1)\right\}M_A^{-2} - \bar{b}(1-K_p)(\Gamma-1)\right]\beta + \left\{\bar{b}(1-K_p)(\Gamma-1) + Z_a\right\}M_A^{-2} = 0, \tag{22}$$

and $\bar{b} = b\rho_1$. Also, the Alfven Mach number M_A is defined by

$$M_A^2 = \frac{\rho_1 U^2}{\mu h_1^2}. \tag{23}$$

Equation (6) together with (9) gives

$$\frac{p}{p_2} = \frac{\rho(1-Z_2)[1+b\rho(1-K_p)]}{\rho_2(1-Z)[1+b\rho_2(1-K_p)]}. \tag{24}$$

3. SELF-SIMILARITY TRANSFORMATION-

The similarity transformation for the problem under consideration are taken as (Vishwakarma and Nath [15])

$$u = U\,V(\eta), \quad \rho = \rho_1 D(\eta),$$
$$p = \rho_1 U^2 P(\eta), \quad Z = Z_1\, D(\eta),$$

$$\mu^{\frac{1}{2}} h = \rho_a^{\frac{1}{2}} U H(\eta), \tag{25}$$

where V, P, H, and D all are functions of non-dimensional similarity variable η only.

The variable $\eta = \frac{r}{R} = \frac{r}{B\exp(\lambda t)}$. (26)

η assumes the value '1' at the shock front and η_p on the piston position.

From equation (24) with aid of equation (25) and (21), we have

$$P(\eta) = \frac{(1-\beta)(\beta-Z_1)D\left[1+\bar{b}D(1-K_p)\right]\left[\beta^2 - \frac{1}{2}M_A^{-2}(1+\beta)\right]}{(1-Z_1 D)\beta\left[\beta+\bar{b}(1-K_p)\right]}. \tag{27}$$

By use of similarity transformations (25) equations (2) to (5) ,can be transformed as

$$(V-\eta)\frac{dD}{d\eta} + D\frac{dV}{d\eta} + \frac{jDV}{\eta} = 0, \tag{28}$$

$$(V-\eta)\frac{dH}{d\eta} + H\frac{dV}{d\eta} + (j-1)\frac{HV}{\eta} + H = 0, \tag{29}$$

$$(V-\eta)\frac{dV}{d\eta} + \frac{H}{D}\frac{dH}{d\eta} + M\frac{dD}{d\eta} + V + \frac{H^2}{D\eta} = 0, \tag{30}$$

where,

$$M = M(\eta) = \frac{(1-\beta)(\beta-Z_1)\left[1+\bar{b}D(1-K_p)(2-Z_1 D)\right]\left[\beta^2 - \frac{1}{2}M_A^{-2}(1+\beta)\right]}{(1-Z_1 D)^2\beta D\left[\beta+\bar{b}(1-K_p)\right]}.$$

Solving equations (28) to (30), we get

$$\frac{dD}{d\eta} = \frac{XD}{\eta}, \tag{31}$$

$$\frac{dD}{d\eta} = \frac{-(V-\eta)X - JV}{\eta}, \quad (32)$$

$$\frac{dH}{d\eta} = \frac{H+HX}{\eta}, \quad (33)$$

where, $X=X(\eta) = \frac{jDV(V-\eta)-VD\eta-2H^2}{[(H^2+MD^2)-D(V-\eta)^2]}$.

Also, by using similarity transformations (25), the shock boundary conditions (21) take the form

$$V(1) = (1-\beta),$$

$$D(1) = \frac{1}{\beta},$$

$$P(1) = \left[(1-\beta) + \frac{M_A^{-2}}{2}\left(1 - \frac{1}{\beta^2}\right)\right],$$

$$H(1) = \frac{M_A^{-1}}{\beta}. \quad (34)$$

The condition that to be satisfied at the piston surface is that the velocity of the fluid is equal to the velocity of piston.

The kinematic condition from equation (23) can be written as

$$V(\eta_p) = \eta_p. \quad (35)$$

The ordinary differential equations (31)to (33) can be numerically integrated with shock boundary conditions (34) to obtain the solution of the problem.

4. ADIABATIC FLOW-

In this section, we present the similarity solution for the adiabatic flow of a perfectly conducting mixture of a non-ideal gas and small solid particles behind a strong shock driven by a cylindrical or spherical piston moving according to an exponential law in the presence of an azimuthal magnetic field.

Here, the shock conditions are the same as the shock conditions (19) of the isothermal flow.

For adiabatic flow, the equations of motion are the equations (3), (4), (5), and equation (Vishwakarma [29], Steiner and Hirschler [30], Vishwakarma and Nath [15])

$$\frac{\partial E_m}{\partial t} + u\frac{\partial E_m}{\partial r} - \frac{p}{\rho^2}\left\{\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right\} = 0. \quad (36)$$

By use of similarity transformations (23), equations (3), (4), (5) and equation (36) can be transformed as

$$(V-\eta)\frac{dD}{d\eta} + D\frac{dV}{d\eta} + \frac{jDV}{\eta} = 0, \quad (37)$$

$$(V-\eta)\frac{dH}{d\eta} + H\frac{dV}{d\eta} + (j-1)\frac{HV}{\eta} + H = 0, \quad (38)$$

$$(V-\eta)\frac{dV}{d\eta} + \frac{H}{D}\frac{dH}{d\eta} + \frac{1}{D}\frac{dP}{d\eta} + V + \frac{H^2}{D\eta} = 0, \quad (39)$$

$$\frac{dP}{d\eta} - \frac{PN}{D}\frac{dD}{d\eta} + \frac{2P}{(V-\eta)} = 0, \quad (40)$$

where

$$N = N(\eta) = \frac{P\left[1+\bar{b}D(1-K_p)(2-z_1D)+(\Gamma-1)\{1+\bar{b}D(1-K_p)\}^2\right]}{D(1-Z_1D)\{1+\bar{b}D(1-K_p)\}}.$$

Solving equations (35) to (38), we get

$$\frac{dD}{d\eta} = \frac{YD}{\eta}, \quad (41)$$

$$\frac{dV}{d\eta} = \frac{-(V-\eta)Y - jV}{\eta}, \quad (42)$$

$$\frac{dH}{d\eta} = \frac{H+HY}{\eta}, \quad (43)$$

$$\frac{dP}{d\eta} = \frac{-NYP}{\eta} - \frac{2P}{(V-\eta)}, \quad (44)$$

where

$$Y = Y(\eta) = \frac{\frac{2P\eta}{(V-\eta)} + jDV^2 - (J+1)VD\eta - 2H^2}{(H^2+ND) - D(V-\eta)^2}.$$

Here, the transformed shock boundary conditions and the kinematic condition at the piston will be same as in the case of isothermal flow (equations (34) and equation (35)).

Now for obtaining the solution to the adiabatic flow, we can numerically integrate the equations (41) to (44), with shock boundary conditions (34).

5. RESULTS AND DISCUSSION-

To obtain the solutions of differential equations (28) to (30) in isothermal flow and (41) to (44) in adiabatic flow, we use numerical integration by Runge - Kutta method of order four along with the shock boundary conditions (35). We start numerical integration from shock front ($\eta = 1$) and continue it until a value η_p (piston position) is approached, where

$$V(\eta_p) = \eta_p.$$

For the purpose of numerical integration the values of constant parameters are taken to be (Pai et al. [2], Miura and Glass[3], Vishwakarma[29], Steiner and Hirschler [30]) $j = 2$; $\gamma = \frac{5}{3}$; $\beta' = 0.25$; $K_p = 0, 0.2$; $G_1 = 1, 100$; $\bar{b} = 0, 0.1$; $M_A^{-2} = 0,\ 0.005,\ 0.01$. The value $j = 2$ corrensponds to spherical shock, $K_p = 0$ to the dust free case, $K_p = 0,\ \bar{b} = 0$ to the perfect gas case, $\bar{b} = 0.1$ to the non-ideal gas, $M_A^{-2} = 0$ to the non-magnetic case. $\beta' = 0.25$ is typical value of ratio of specific heat of small solid particles and specific heat of gas at constant volume.

Table 1: Values of the density ratio β across the shock front and the position of piston η_p at different values of parameters K_p, G_1 , $\bar{b}$ and M_A^{-2} for $j = 2$, $\gamma = 5/3$ and $\beta' = 0.25$.

$M_A{-}2$	$\bar{b}$	G_a	K_p	β	η_p	
					Isothermal flow	Adiabatic flow
0	0	1	0	0.25	0.92989	0.945032
0	0	1	0.2	0.391045	0.86943	0.88141
0	0	100	0.2	0.2407042	0.93211	0.94723
0	0.1	1	0	0.3065522	0.90925	0.92381
0	0.1	1	0.2	0.4176798	0.86	0.87182
0	0.1	100	0.2	0.2879469	0.91488	0.92969
0.005	0	1	0	0.255571	0.92789	0.94202
0.005	0	1	0.2	0.3925079	0.86938	0.88112
0.005	0	100	0.2	0.2465791	0.93004	0.94411
0.005	0.1	1	0	0.309912	0.90833	0.92243
0.005	0.1	1	0.2	0.4189384	0.86	0.87158
0.005	0.1	100	0.2	0.2917437	0.91381	0.92809
0.01	0	1	0	0.2610386	0.92599	0.93934
0.01	0	1	0.2	0.3940022	0.86932	0.88082
0.01	0	100	0.2	0.252331	0.92808	0.94133
0.01	0.1	1	0	0.3132893	0.90741	0.92105
0.01	0.1	1	0.2	0.4202221	0.86	0.87134
0.01	0.1	100	0.2	0.2955502	0.91273	0.92651

Table -1 shows the values of density ratio β across the shock front and the value of piston position η_p in both the cases, when the flow is isothermal and adiabatic, for different values of the parameters K_p, G_1, $\bar{b}$ and M_A^{-2}. It is clear from Table-1 that the piston position is significantly affected due to the presence of dust particles; whereas little affected due to the presence of magnetic field.

Figures 1(a, b, c, d) and 2(a, b, c, d) show the variation of flow variables with respect to dimensionless variable η, in the flow-field behind the shock in the isothermal and adiabatic flows respectively. It is clear from these figures that values of non-dimensional velocity V, non-dimensional density D, non-dimensional pressure P and non-dimensional magnetic field H increase in general as we move from shock front towards the piston. Figure 2(b) shows that there is an unbounded density distribution near the piston in the adiabatic flow.

Effects of an increase in the value of M_A^{-2}(strength of ambient magnetic field) are

(i) to increase the value of β i.e. to decrease the shock strength (see Table 1);

(ii) to decrease η_p i.e. to increase the distance of piston from the shock front (see Table 1);

(iii) to decrease the flow-variables V, D and P at any point in the flow field behind the shock (see Figures 1(a, b, c) and 2(a, b, c)); and

(iv) to increase the magnetic field H in the flow-field behind the shock (see Figures 1(d) and 2(d)).

Effects of an increase in the value of mass concentration of solid particles K_p are

(i) to increase the value of β at $G_1 = 1$, but to decrease at $G_1 = 100$ i.e. to decrease the shock strength at $G_1 = 1$, but to increase at $G_1 = 100$ (see Table1);

(ii) to decrease η_p at $G_1 = 1$, but to increase at $G_1 = 100$ (see Table 1);

(iii) to decrease the flow variables V, D and H at $G_1 = 1$ but to increase at $G_1 = 100$ at any point in the flow-field behind the shock (see Figures 1(a, b, d) and 2 (a, b, d)); and

(iv) to decrease the pressure P at $G_1 = 1$, and to increase at $G_1 = 100$.

When $G_1 = 1$, the density of small solid particles is equal to the density of the non-ideal gas in the mixture. Thus in the case of $G_1 = 1$, small solid particles occupy a significant portion of the volume of the mixture which cause a decrease in the compressibility of the mixture remarkably. Also when $G_1 = 1$, $Z_1 = K_p$, then an increase in K_p gives an increase in Z_1(the volume fraction of solid particles) which causes a further decrease in the compressibility and hence it results in increase in the distance between the shock and piston and a decrease in the shock strength. Now when $G_1 = 100$, the density of small solid particles is equal to hundred times to the density of the gas and so in this case, small solid particles occupy a very small portion of the volume. Hence it results in small decrease in the distance between the shock and piston and a increase in the shock strength.

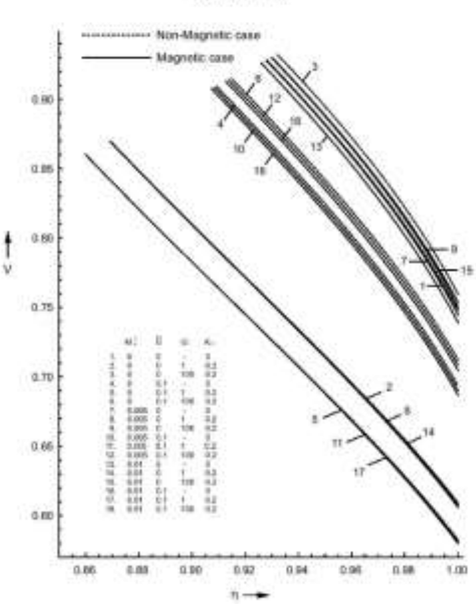

Figure 1(a): Variation of velocity in the region behind the shock front.

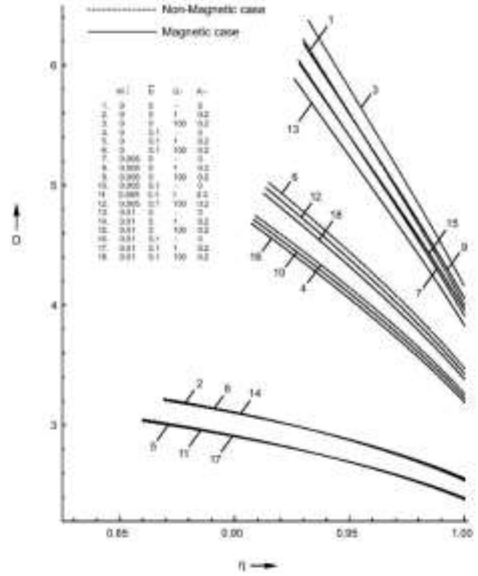

Figure 1(b): Variation of density in the region behind the shock front.

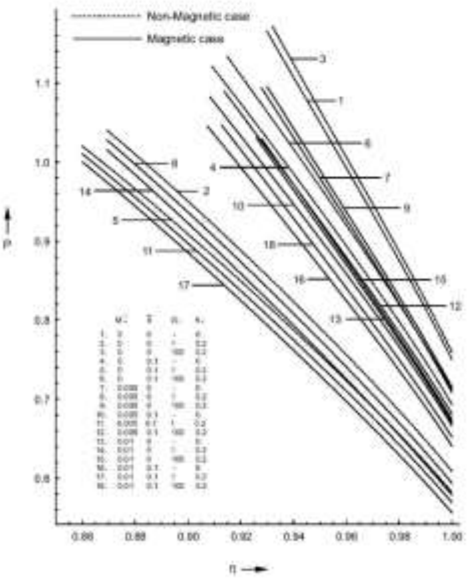

Figure 1(c): Variation of pressure in the region behind the shock front.

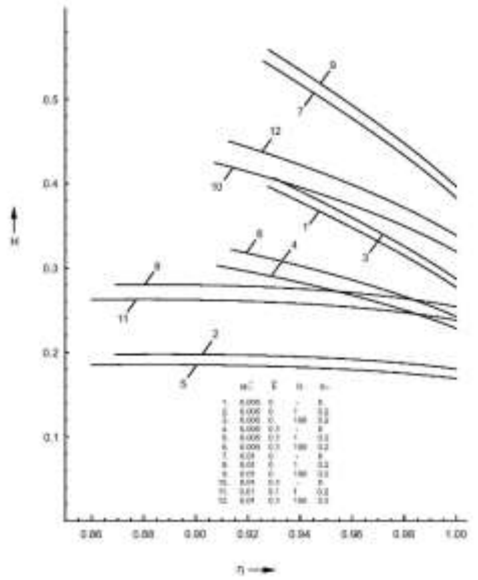

Figure 1(d): Variation of magnetic field in the region behind the shock front.

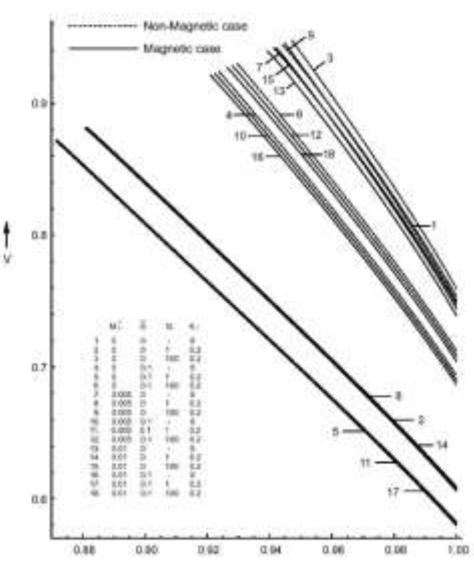

Figure 2(a): Variation of velocity in the region behind the shock front.

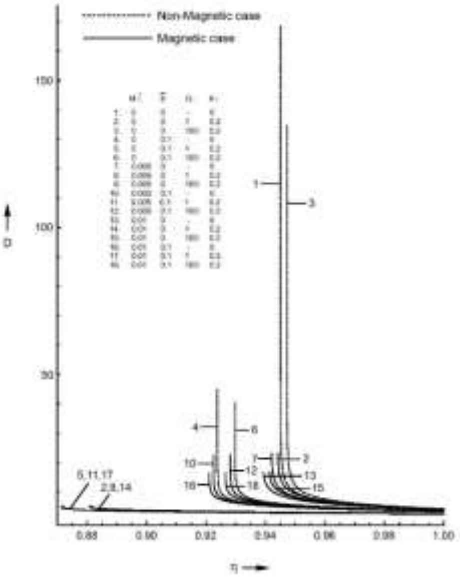

Figure 2(b): Variation of density in the region behind the shock front.

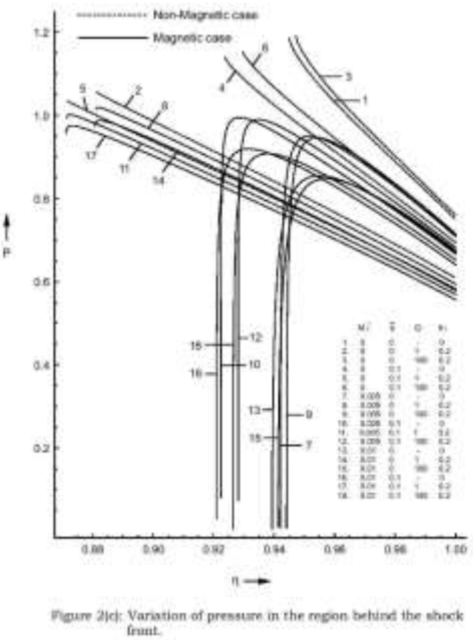

Figure 2(c): Variation of pressure in the region behind the shock front.

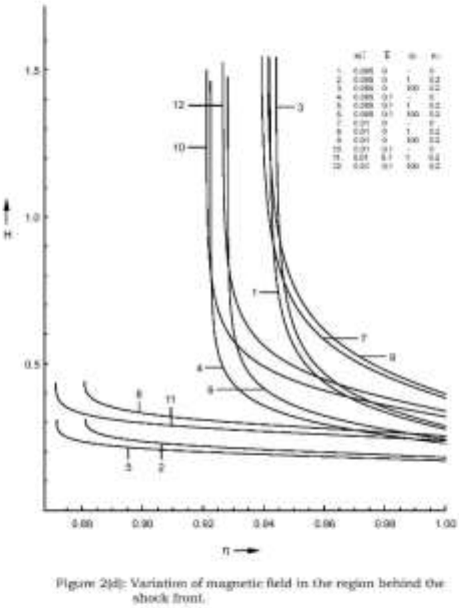

Figure 2(d): Variation of magnetic field in the region behind the shock front.

Effects of an increase in the value of G_1 are

(i) to decrease the value of β i.e. to increase the shock strength (see Table 1);
(ii) to increase η_p (see Table 1);

(iii) to increase the flow-variables V, D, P and H (see Figures1(a, b, c, d) and 2(a, b, c, d)).

Effects of an increase in the value of non-idealness parameter $\bar{b}$ are

(i) to increase the value of β i.e. to decrease the shock strength, (see Table 1);

(ii) to decrease η_p (see Table 1);

(iii) to decrease the flow-variables V, D, P and H (see Figures1(a, b, c, d) and 2(a, b, c, d).

An increase in the non-idealness parameter $\bar{b}$, the internal volume occupied by the gas molecules increases, resulting in to in the compressibility of the mixture.

Mutual effects of parameters K_p, G_1, $\bar{b}$ and M_A^{-2} on the shock strength ($\frac{1}{\beta}$), on the piston position (η_p) and on the profiles of the flow-variables V, D and P :

(i) The effects of the presence of dust particles and the effects of non-idealness of gas on the shock strength and on the piston position are reduced due to the presence of magnetic field. Actually $\frac{1}{\beta}$ and η_p decrease by increasing $\bar{b}$, whereas increase by increasing G_1 ; also $\frac{1}{\beta}$ and η_p decrease by increasing K_p when $G_1 = 1$. These effects of K_p, G_1 and $\bar{b}$ on $\frac{1}{\beta}$ and η_p are reduced by increasing the value of M_A^{-2}.

(ii) The effects of K_p, G_1 and $\bar{b}$ on the profiles of the flow-variables V, D and P are not affected due to the presence of magnetic field.

(iii) The effects of magnetic field and the effects of non-idealness of the gas on the shock strength, piston position and on the flow-variables V, D and P are significantly reduced due to the presence of dust particles. Actually $\frac{1}{\beta}$, η_p and flow-variables V, D and P decrease by increasing M_A^{-2} as well as by increasing $\bar{b}$, but these effects of M_A^{-2} and $\bar{b}$ are profoundly reduced by increasing the value of K_p .

Comparison between the isothermal and adiabatic flows of the mixture:

(i) The density is finite at the piston in the flow-field behind the shock front, in the case of isothermal flow, whereas in the case of adiabatic flow it becomes unbounded at the piston. (see Figures 1(b) and 2(b)).

The above difference between the densities of the two flows seems to be necessary because with an unbounded density distribution near the piston in the case of isothermal flow violates the assumption of zero temperature gradient.

(ii) The piston position η_p is greater in the case of adiabatic flow in comparison with that in the case of isothermal flow, i.e. distance between the shock front and piston is less in the case of adiabatic flow in comparison with that in the case of isothermal flow (see table 1).

(iii) The pressure rapidly decreases to zero at the piston when the flow is dust free or almost dust free ($G_1 = 100$, $K_p = 0.2$) in the case of adiabatic flow, whereas it remains finite at the piston in the case of isothermal flow (see Figures 1(c) and 2(c)).

6. CONCLUSION:

Present work, investigates the self-similar flow of a perfectly conducting mixture of a non-ideal gas and small solid particles, behind a strong shock driven out by a piston moving according to an exponential law, in the presence of an azimuthal magnetic-field for both the cases when the flow is isothermal and adiabatic. It is observed that the shock strength ($\frac{1}{\beta}$), value of piston position (η_p) and also the effects of dust parameters K_p, G_1 and the non-idealness parameter of the gas $\bar{b}$ on the shock strength and on the piston position are little affected due the presence of magnetic field. It is also observed that shock strength, value of piston position and the effects of magnetic field and effects of non-idealness of the gas on $\frac{1}{\beta}$, on η_p and on the profiles of the flow-variables are significantly affected due the presence of dust particles.

These effects can be illustrated as-

(i) The shock strength ($\frac{1}{\beta}$) and the value of piston position (η_p) slightly decrease due to the presence of magnetic-field.

(ii) The effects of parameters K_p, G_1 and $\bar{b}$ on $\frac{1}{\beta}$ and on η_p are slightly reduced due to the presence of magnetic-field.

(iii) The shock strength, the value of piston position and effects of parameters M_A^{-2} and $\bar{b}$ on $\frac{1}{\beta}$, on η_p and on the profiles of the flow -variables V, D and P are significantly reduced due to the presence of dust particles.

It is further demonstrated that the assumption of zero temperature gradient brings a profound change in the density distribution as compared to that in the adiabatic flow whereas other flow-variables are little affected.

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